JUNIOR PROBLEMS

Solutions to the problems stated in this issue should arrive before November 5, 2016.

Proposals

56. *Proposed by Valmir Krasniqi, University of Prishtina, Republic of Kosova.* Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that $f(m! + n!) | f(m)! + f(n)!$ and $m + n$ divides $f(m) + f(n)$ for all $m, n \in \mathbb{N}$.

57. *Proposed by Angel Plaza, University of Las Palmas de Gran Canaria, Spain.* Let $x_1, x_2, ..., x_n > 0$. Prove that

$$\left(\frac{\sum_{k=1}^{n} x_k}{n}\right)^2 \leq \frac{1}{n} \sum_{k=1 \text{ cyclic}}^{n} \frac{x_k^2 + x_k x_{k+1} + x_{k+1}^2}{3} \leq \frac{\sum_{k=1}^{n} x_k^2}{n}.$$

58. *Proposed by Arkady Alt, San Jose, California, USA.* Let P be arbitrary interior point in a triangle ABC and r be inradius. Prove that

$$\frac{a^2}{d_a(P)} + \frac{b^2}{d_b(P)} + \frac{c^2}{d_c(P)} \geq 36r^2$$

if $d_a(P), d_b(P), d_c(P)$ be distances from the point P to sides BC, CA, AB respectively.

59. *Proposed by Marcel Chiriţă, Bucharest, Romania.* Solve in real numbers the system

$$\left.\begin{array}{l} 2^x + 2^y = 12 \\ 3^x + 4^z = 11 \\ 3^y - 4^z = 25 \end{array}\right\}.$$

60. *Proposed by Dorlir Ahmeti, University of Prishtina, Department of Mathematics, Republic of Kosova.* Let ABC be an acute triangle. Let D be the foot of the altitude from A. Let E, F be the midpoints of AC, AB, respectively. Let $G \neq B$ and $H \neq C$ be the intersection of circumcircle of the triangle ABC with circumcircles of the triangles BFD and CED, respectively. Suppose that A, G, B, H, C are order in this way on the circle they belong. Show that line EF, HB and CG are concurrent.